Faculty: Science and Technology

Course: **Measure Theory**

Programme: Study Abroad in Engineering

Semester: 1 - Fall

ECTS credits: 6

Duration: 45 hours

Language of instruction: English

Instructor: Nora Silva

Course Description

The objective of this elective is to give engineering students the scientific skills necessary to model and analyze random phenomena. The student will become familiar with the manipulation of the Lebesgue integral and its use in both analysis and probability.

Prerequisites

Probability and Analysis courses.

Attendance policy

Attendance is mandatory for all classes, including study visits. Any exams, tests, presentations, or other work missed due to student absences can only be rescheduled in cases of certified medical or family emergencies.

Learning outcomes

The student will become familiar with the manipulation of the Lebesgue integral and its use in both analysis and probability.

Method of presentation

- Lectures and discussions: Lectures with appropriate visual support provide the theoretical content of the sessions. Class discussions facilitate the students' ability to connect reading and lectures, analyzing or applying concepts.
- Class participation: Students are expected to participate in group activities and in the discussions based on the course readings and cases proposed.
- Home exercises: Students are expected to solve several exercises during the semester.

Required work and assessment methods

- Challenges (20%) and test (40%: 20/12/2023). Preparation, development and discussions related to exercises will be highly valuable for the success of the course both at individual andgroup level.

- Project (30%) and participation (10%). Individual active participation in discussions, a final project, oral presentations and team work. The positive and proactive attitude of the student will be encouragedand valued by instructors throughout the course.

Retake exams and activities

The student must present the activities, challenges and exercises pending to delivery. If the student passed the test, it is not necessary to take the retake test. It is only mandatory to take the retake test if the test were not passed.

The activities weight in the retake evaluation is the same as activities weight in the ordinary evaluation, but then the maximum grade will be 5.

Contents

Unit One: Borel's Normal Number Theorem

Week 1. The unit interval. Borel's Normal Number Theorem.

Unit Two: Probability Measures and Measures

Week 2 - 3. Spaces. Classes of sets. Lebesgue measure in the unit interval.

Unit Three: Existence and Extension

Week 4. Construction of the extension. Monotone classes. Nonmeasurable sets.

Unit Four: Denumerable Probabilities

Week 5. General formulas. The Borel-Cantelli Lemmas.

Unit Five: Simple Random Variables

Week 6-7. Definition. Convergence of random variables. Independence. Expected value.

Unit Six: The Law of Large Numbers

Week 8 - 9. The strong law. The weak law. Bernstein's Theorem.

Unit Seven: Functions and Integrals

Weeks 10 - 11. Measurable functions. Properties that hold almost everywhere. Limit theorems. Lebesgue integral. Fatou's lemma.

Unit Eight: Convergence

Weeks 12. Modes of convergence.

Unit Nine: Product Measures

Weeks 13. Constructions. Fubini's Theorem. Applications.

Recommended reading

P. Billingsley, Probability and Measure (3rd Ed.). Wiley, New York.

Measure Theory. D.L. Cohn (2nd Ed.). Birkhäuser.